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Minimization of Numerical Dispersion Errors in Finite Element Models of Non-homogeneous Waveguides

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Abstract. The paper presents the approach for the reduction of numerical errors, which are inherent for simulations based on wave propagation models in discrete meshes. The discrete computational models always tend to generate errors of harmonic wave propagation velocities in higher frequency ranges, which can be treated as numerically-induced errors of dispersion curves. The result of the errors is the deterioration of the shapes of simulated waves as the time of simulation increases. The presented approach is based on the improvement of the matrices of elements of the finite element model by means of correction of the modal frequencies and modal shapes of an individual element. The approach developed by the authors earlier and proved to work in the case of a uniform waveguide now has been demonstrated to be valid for simulations of waves in networks of waveguides. The non-reflecting boundary conditions can be implemented by combining synthesized and lumped mass elements in the same model. The propagating wave pulses can be satisfactorily simulated in comparatively rough meshes, where only 6-7 finite elements per wavelength are used.

Keywords: finite elements, wave propagation, modal synthesis, modal errors.

1 Introduction

The short-waves and pulses propagation simulations are encountered in various engineering applications, such ultrasonic measurement techniques oriented for defects and impurities detection inhomogeneous structures, hydraulic pressure pulses propagation in large pipeline networks, etc. The concept of the short-wave considered in this paper concept relies on the comparison of the length scales of the shapes of propagating waves and of the model of the propagation environment. We assume that the wavelength is hundreds or thousands times shorter than the characteristic length of the propagation environment. One of the most important problems, which occur in designing and implementing finite element (FE) models of the wave propagation, is huge dimensionalities of the models and there for every high demands for computing resources. This is exceptionally important in simulations of short waves propagation. In order to achieve areas on able accuracy of the computation extremely dense finite element meshes are necessary. A highly refined FE mesh in its turn requires very small time integration steps, which increase the computation time even more.

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Generally, the dimensionality of the computational models is reduced as rougher meshes are applied. The measure for roughness of the mesh is the number of elements per characteristic wavelength. Unfortunately, rough meshes tend to increase the simulation errors, which exhibit themselves as severe deterioration of the shapes of propagating wave pulses as the time of simulation increases. It is well known that this happens due to the errors of representation of wave propagation velocities of different harmonic components of the propagating pulse. In any discrete model of wave propagation waves of different frequencies propagate with slightly different velocities than they should in reality, and the magnitude of an error depends on the frequency of the wave. The relationship of the wave velocity against the wave frequency or against the wavelength is called the dispersion curve, therefore the errors under consideration are often referred to as numerical dispersion errors or phase velocity errors.

Already in early 1980 it was noticed that solutions provided by the models using lumped and consistent mass matrices tend to generate essentially different patterns of the deterioration of propagating wave pulses because of errors of modal frequencies of the individual finite elements of the structure [1]. The weighted average of the consistent and lumped mass matrices is referred to as the generalized mass matrix, which enables to obtain same accuracy of the overall model with less number of elements per wavelength. In 2004 the element matrix synthesis technique was proposed for modification of modal shapes and frequencies of an individual element such that after assembling the element matrices into structural matrices the overall model would generate minimal possible phase velocity errors [2]. The performance of the method was examined in the case of 1D homogenous structures consisting of coextensive elements. About 80% of the natural frequencies of the overall model generated errors less than 2% compared with exact solution, and only 6-7 elements per wavelength were enough to obtain same accuracy as in models based on generalized mass matrices with 17-18 nodes per wavelength. In [3] it was demonstrated that for 2D homogenous structures assembled of identical elements based on synthesized mass matrices the method worked properly and maintained similar accuracy. In order to analyze parts of large models as particular sub-models the non-reflecting boundary conditions at the cut boundaries were applied [4].

In this paper the performance of the models based on the synthesized matrices was examined in the case of wave propagation in non-homogeneous branched 1D structures. As a sample structure the FE model for transient pressure wave simulation in fluid pipe network was constructed. Several finite element models suited for fluid and gas flow transients have been reported. In [5] the finite element model of the flow in the pipe with laminar frequency-dependent friction was developed. In [6] the formulation of the fluid-structure interaction included axial vibration model of the pipe aiming at better estimating the relative velocity of the fluid against the pipe wall. Most reported finite element models of transient pipe flow were based on simplified systems of governing equations, in which convection and (or) non-linear terms can be neglected. A characterization of different options of existing transient models and approaches to their solution has been provided in [7]. The approach developed in this work treats the finite element model of the sample pipe work structure as a standard structural dynamic equation in terms of pressure variables and their first and second-order time derivatives, where the flow velocities compared with pressure wave speed are very small and can be neglected. The results obtained in his work justify the validity of the synthesized matrices approach in the case of non-homogeneous structures, as well as reveals certain difficulties when examining the non-reflecting boundary conditions.

2 Modal Synthesis Technique

A linear dynamic finite element model used for simulation of vibrations and wave propagation can be always presented as

$$[\mathrm{M}]\{\ddot{\mathrm{U}}\} + [\mathrm{C}]\{\dot{\mathrm{U}}\} + [\mathrm{K}]\{\mathrm{U}\} = \{\mathrm{R(t)}\} \tag{1}$$

where $[M]$, $[C]$, $[K]$ are structural mass, damping and stiffness matrices, $\{U(t)\}$ is the nodal displacement vector, $\{R(t)\}$ is the lumped forces vector.

By assuming that the damping forces are very small the proportional form of the damping matrix as $[\mathrm{C}] = \mathrm{a}[\mathrm{M}]$ is employed. The form of equation (1) is the same for an individual element, as well as, for the finite element structure, the only difference being the dimensionality of vectors and matrices.

Mass and stiffness matrix could be expressed using modal synthesis technique

$$[\mathrm{M}] = ([\mathrm{Y}]^{\mathrm{T}})^{-1}[\mathrm{Y}]^{-1} \tag{2.1}$$

$$[\mathrm{K}] = ([\mathrm{Y}]^{\mathrm{T}})^{-1}[\mathrm{diag}(\omega_1, \omega_2, \dots, \omega_n)][\mathrm{Y}]^{-1} \tag{2.2}$$

where $[Y]$ and $diag(\omega_1, \omega_2, \dots, \omega_n)$ are shape functions and modal frequencies of non damped structures. Synthesized matrices of elements $[\widetilde{M}_{el}]$ and $[\widetilde{K}_{el}]$ obtained by properly modifying their modal frequencies and modal shapes. The goal of the modification is that the computational domains assembled of the synthesized element matrices would generate minimal possible phase velocity errors. It accomplished by taking first N exact modal shapes and modal frequencies of a domain and modifying them by means of properly selected coefficient vectors $\{a^{\omega}\}$ and $\{a^{y}\}$ as

$$[\mathrm{diag}(\widetilde{\omega}_1, \widetilde{\omega}_2, \dots, \widetilde{\omega}_N)] = [\mathrm{diag}(\omega^2)]\{\mathrm{a}^{\omega}\} \tag{3.1}$$

$$[\{\tilde{y}_1\}, \{\tilde{y}_2\}, \dots, \{\tilde{y}_N\}] = [\mathrm{Y}]\{\mathrm{a}^{y}\} \tag{3.2}$$

where $\{\tilde{y}_1\}, \{\tilde{y}_2\}, \dots, \{\tilde{y}_N\}$ and $diag(\omega_1, \omega_2, \dots, \omega_n)$ are synthesized element shape functions and modal frequencies. The first N nearly exact modal shapes and frequencies can be obtained by presenting the volume occupied by an individual element by means of a highly refined model of dimensionality $n.$N. Coefficients $\{\mathrm{a}^{\omega}\}$ and $\{\mathrm{a}^{y}\}$ are computed by minimizing modal frequency error as target function

$$\min_{\{\mathrm{a}^{\omega}\},\{\mathrm{a}^{y}\}} \Psi = \sum_{\mathrm{i}=1}^{\mathrm{N}} \left(\frac{\widehat{\omega}_{\mathrm{i}} - \widehat{\omega}_{\mathrm{i0}}}{\widehat{\omega}_{\mathrm{i0}}}\right)^2 \tag{4}$$

where $\widehat{\omega}_i$– modal frequency of i-th mode of domain assembled of synthesized elements, $\widehat{\omega}_{i0}$– exact value of the modal frequency of i-th mode. The modal frequency error of the joined domain is minimized by employing the gradient descend method, where sensitivity functions $\frac{\partial\Psi}{\partial\{a^y\}}$ and $\frac{\partial\Psi}{\partial\{a^\omega\}}$ are employed.

3 A Pressure Impulse Propagation FE Model as an Example of a Branched 3D Structure of Uni-dimensional Waveguides

The basic set of uni-dimensional flow equations, which contains the continuity equation and the linear momentum conservation equation reads as

$$\begin{cases} \dfrac{\delta m}{\delta t} + \dfrac{\delta(mv)}{\delta x} = 0; & (5.1) \\ \dfrac{\delta(mv)}{\delta t} + \dfrac{\delta(mv^2)}{\delta x} + A\dfrac{\partial p}{\partial x} + \dfrac{f}{D}\dfrac{mv|v|}{2} + mg\sin a = 0. & (5.2) \end{cases}$$

where δx – differential element of the fluid in the pipe of uniform cross-section, p – is the fluid pressure, $m = \rho A$ is the mass of the fluid of mass density ρ per unit length of the pipe of cross-sectional area A, v is the velocity of the fluid flow, a is the angle of the pipe to against the horizontal, g is the free-fall acceleration, $f = \frac{0.3614}{RE}$ is the friction coefficient for turbulent flow, where $RE = \frac{\rho D|v|}{\mu}$ is the Reynolds number (μ – the dynamic viscosity of the fluid). The standard FE model for pipeline flow cannot be directly gained from (5) equation. At low values of flow velocity compared to pressure wave propagation speed and nearly incompressible fluid, equation (5) can be expressed as a standard structural dynamic equation without non-linear and convection terms as

$$\begin{cases} \dfrac{\delta^2 p}{\delta t^2} + \dfrac{f}{D}|v|\dfrac{\delta p}{\delta t} - \dfrac{\widetilde{K}}{\rho_0}\dfrac{\delta^2 p}{\delta x^2} = 0; \\ \dfrac{\delta v}{\delta t} = -\dfrac{1}{\rho_0}\dfrac{\delta p}{\delta x} - \dfrac{f}{D}\dfrac{v|v|}{2} - g\sin a. \end{cases} \qquad (6)$$

where $\widetilde{K} = K(1 + \frac{KD}{hE})^{-1}$ is the equivalent bulk modulus of the pipe, which combines the bulk stiffness of the fluid and the radial expansion stiffness of the pipe, K is the bulk modulus of the fluid, E is the stiffness modulus of the pipe material, D and h are the diameter and the wall thickness of the pipe.

In this paper we examine pressure wave propagation in non-damped systems where and dynamic equation of (6) can be transform into equation (1), where the element matrices read as

$$[\mathrm{M^e}] = \mathrm{A}\int_0^{\mathrm{L}} [\mathrm{N}]^{\mathrm{T}}[\mathrm{N}]\mathrm{d}x \approx \frac{\mathrm{AL}}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \approx \frac{\mathrm{AL}}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{7.1}$$

$$[\mathrm{K^e}] = \frac{\mathrm{A}}{\mathrm{L}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{7.2}$$

$$\{\mathrm{R^e}\} = -\mathrm{A}\widetilde{\mathrm{K}}\left(\frac{\mathrm{p}^*}{\mathrm{L}} - \mathrm{g}\sin \mathrm{a}\right)\begin{Bmatrix} 1 \\ -1 \end{Bmatrix} \tag{7.3}$$

$$[\mathrm{C^e}] = 0 \tag{7.4}$$

where in (6.3) p^* – is the pressure created by a pump, if the pump is assumed to work within the region presented by this particular finite element.

The modal synthesis technique described in section 2 of this work is applied to the subdomains formed of elements with matrices (7.1) and (7.2) in order to obtain the synthesized matrices exhibiting the improved dynamic performance.

4 Numerical Investigation

The verification of the model assembled of synthesized elements is carried out by analyzing its behavior in sample situations and by comparing the obtained results against the solutions obtained in highly refined (>35 nodes per wavelength) meshes of elements employing the generalized mass matrices. In this work models of synthesized elements are assembled of 10-node elements, where a^{ω} and a^{y} parameters are computed by solving problem (4) for first 89% modal frequency errors of the refined model. For the analysis of wave propagation in non-homogenous waveguides different types of pipes were used in the sample structures. The integration in time was performed by means of the central difference method. The physical parameters of the model were selected corresponding to the waveguide as the water-filled pipeline with bulk modulus $K = 2.2^{9}(N/m^2)$, mass density $\rho = 995(kg/m^3)$ and dynamic viscosity $\mu = 5.47^{-4}$ $({N * s}/{m^2})$. Evaluations were accomplished by comparing modal frequency errors and the propagating pressure impulse shapes obtained by using the models of synthesized elements and the models of elements based on the generalized mass matrices containing the same number of elements per wave length. Element number N per wavelength was selected experimentally by modeling the pressure pulse in a uniform 2700 (m) length pipe of 91 node with Young's modulus $E = 2.1^{11}$ (N/m^2), diameter $D = 0.1$ (m), wall thickness $h_1 = 0.0035$ (m) which pressure wave speed $C = 1112.7$ (m/s). The excitation pressure pulse was generated at the left hand end of the pipe as the pressure overshoot 10^5 (Pa) as a half-sine pulse of duration $dt = 0.147$ (s).

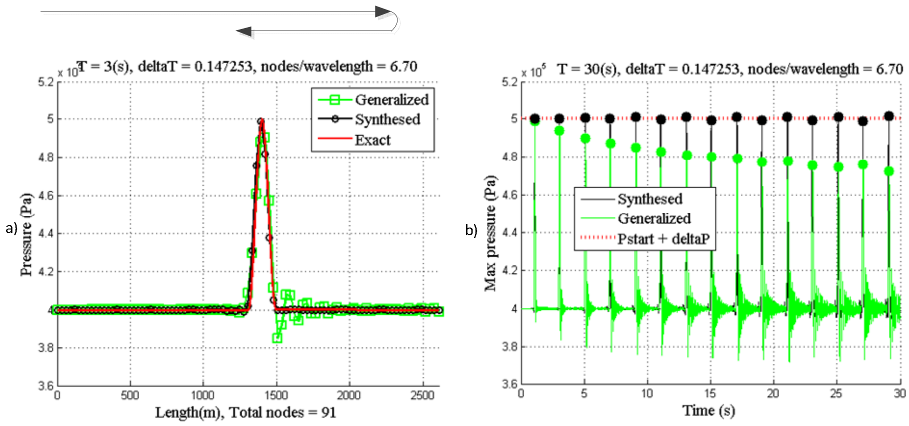

Fig. 1. a) Pressure pulse after 3s. simulation. b) Pressure in 47 node for 30s simulation.

Figure 1a represents the pressure pulse after 3 (s) in the case of 6.7 elements per wavelength. The synthesized model (black line) performs in close coincidence with the exact solution (red line), while the model based on generalized mass matrix elements (green line) generates significant errors in front of the main pulse. As the simulation time is increased, the accumulation of errors is evident. Figure *1b* represents the vibration of a selected node (in this example node 47) during 30 (s) simulation, where small circles mark the time instances as the front of the impulse arrives at the node.

The model of sequentially connected pipes of two different types (diameters $D_1 = 0.1(m)$, $D_2 = 0.05(m)$, wall thickness $h_1 = 0.0035\ (m)$, $h_1 = 0.0025\ (m)$ and Young's modulus $E = 2.1^{11}\ (N/m^2)$) has been employed as an example of the non-homogenous model.

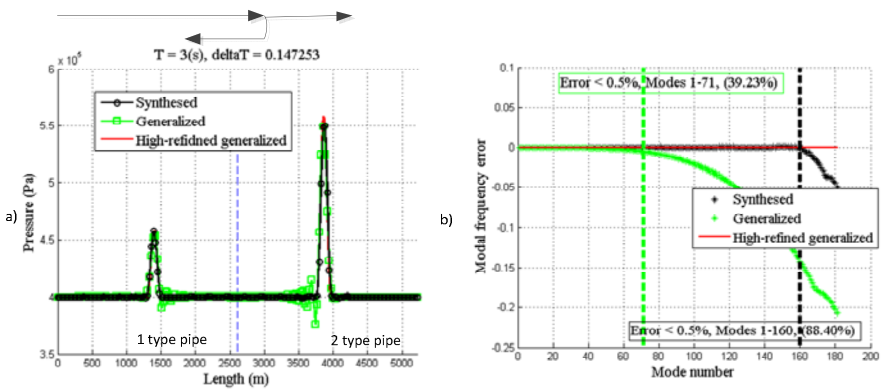

Fig. 2. a) Pressure impulse after 3(s). b) Model frequency error.

Figure 2a represents the pressure pulse shape after $3(s)$, while the pulse was partially reflected at the intersection of the pipes. The model based on synthesized elements, 6.7 elements per wavelength, provided satisfactory results compared with those obtained in a highly refined mesh (~42 nodes per wavelength) model based on generalized mass matrices. The model based on the generalized mass matrices with 6.7 nodes in wavelength generated significant errors due to the numerical dispersion. In Figure 2b modal frequency errors produced by different models are compared, where dashed line marks boundary of the modes region with the modal frequency error less than 0.5%. It can be seen that in the synthesized model the error less than 0.5% is ensured for about 89% of the total number of modes.

The branched non-homogenous model composed of uni-dimensional waveguide segments with non-reflecting boundary conditions has been analyzed (Figure 3a).

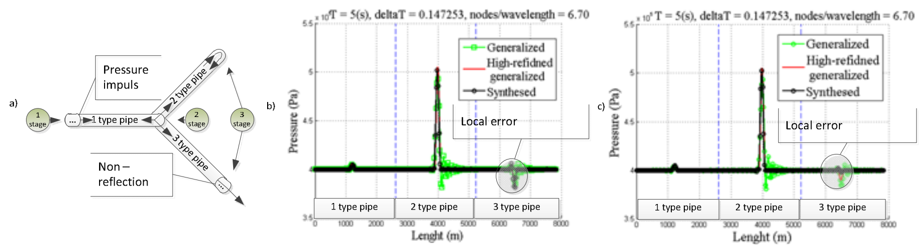

Fig. 3. a) Branched inhomogeneous model b) Local error caused boundary non reflection in model of synthesized elements c) Local error in model with one small lumped element before boundary non reflection condition implementation

There are 3 types pipes (diameters $D_1 = 0.1(m)$, $D_2 = 0.08(m)$, $D_3 = 0.05(m)$, wall thickness $h_1 = 0.0035\ (m)$, $h_2 = 0.003\ (m)$, $h_3 = 0.0025\ (m)$ and Young modulus $E = 2.1^{11}\ (N/m^2)$) used in model. At stage 1 the pressure wave actuated by $10^5(Pa)$ half-sine pulse in the left-hand end of the pipe type 1. At stage 2 (after $2(s)$) the impulse is partially reflected, and distributed to pipes of types 2 and 3. At stage 3 (after $4(s)$) at the end of pipe type 2 the impulse reflects and comes back while at the end of pipe type 3 the non- reflecting boundary condition is implemented. Figure 3b represents the situation after $5(s)$, where pressure impulse has traveled ~$6.8km$ (wave speeds are different in the pipes of different types). As can be seen, the junctions between different segments of the model based on synthesized matrices do not introduce any marked disturbances in the simulation results. This means that the synthesized matrices approach is working properly for non-homogeneous waveguide network models. The non-reflecting boundary condition applied directly to the synthesized matrices model does not work properly and generate significant local error up to 13% of the impulse height. However, the error could be easily reduced up to ~1.5% of impulse amplitude by adding a short-length lumped mass matrix finite element just before the point where the non-reflecting boundary condition was implemented. Thus the overall performance of the model was even better than obtained by the highly refined model based on the generalized mass matrices (Figure *3c*). Local errors could not be completely eliminated. Probably, they were caused by the combination of elements of two different types (synthesized and lumped) within the same model.

5 Conclusion

The approach for the reduction of the simulation errors of propagating pulses in unidimensional waveguides has been investigated in the case of branched non-homogeneous structures with implemented non-reflecting boundary conditions. The overall approach based on synthesized mass matrices has been earlier verified for the case of uniform waveguides. The results of this work demonstrated that the approach is valid for wave propagation simulations in branched structures joined of 1D segments possessing different characteristic wave propagation speeds. Non-reflecting boundary conditions applied directly for synthesized models generate significant errors up to 13% of the pulse height. The addition of a small lumped element just before the non-reflection boundary point enabled to reduce the error significantly, where only a small local error remained. One of possible application areas of the developed model is the simulation of the pipeline leakage monitoring system, the purpose of which is to determine the location of the leakage (pressure drop pulse) based on pressure variations registered by meters arranged at various places of the pipeline.

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